Polydron Sudoku

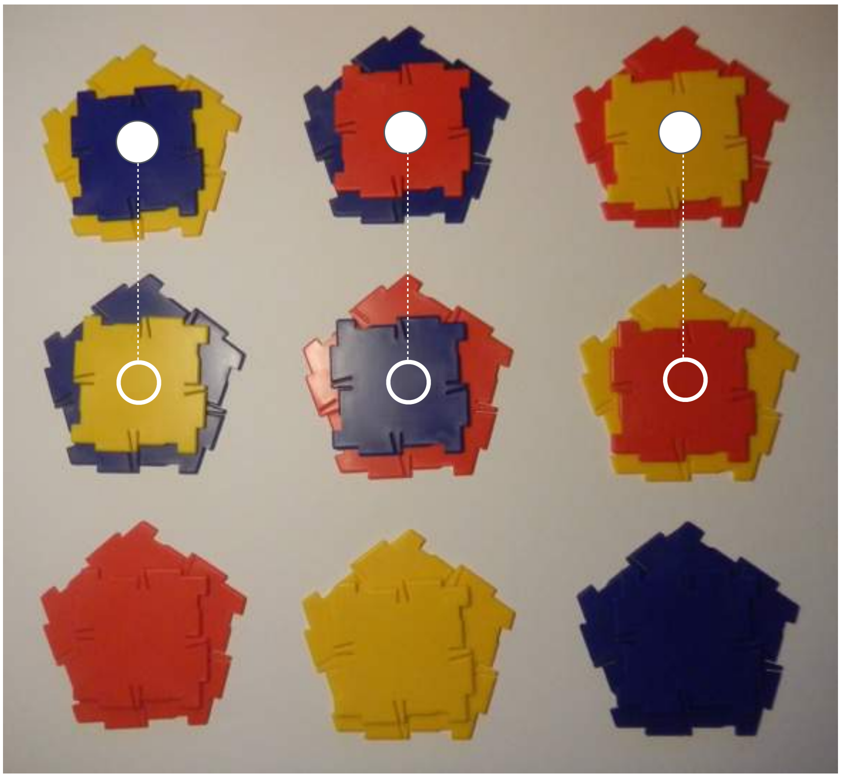
Most Sudoku puzzles use the numbers 1 to 9, but you can use anything – letters, colours, shapes, … And you can make up puzzles with fewer items. We shall use just 4. If you think such puzzles are likely to be too simple, I warn you that we shall move from ‘single’ Sudoku to ‘double’ Sudoku and even ‘triple double’ Sudoku.

I’ll explain what I mean.

In the standard Sudoku puzzle you complete a square with the numbers 1 to 9 so that every number occurs in every row and every column. The negative way to put that is to say that no number occurs twice in any row or any column. Following Leonhard Euler, who studied these things in the eighteenth century, mathematicians call such a square a *Latin* square. Here is an order 3 Latin square, using Polydron pentagons in 3 colours. (Think of these as saucers.)

I call solving Latin Squares doing ‘single’ Sudoku. Not too hard for a 3 x 3 square.

But what if you add cups (Polydron squares) to the saucers so that every saucer row and every saucer column and every cup row and every cup column contain all 3 colours and each cup-saucer pair occurs just once. Again following Euler, we call these *Graeco-Latin* squares and I call solving Graeco-Latin squares doing ‘double’ Sudoku.



Still not too hard. In the picture I’ve joined ‘paired pairs’: the colours are switched between one cup-saucer pair and the other.

So let’s move to an order 4 Latin square:

Experiment with these: there are many colour patterns you can make. But again, not too hard.

So we’ll attempt an order 4 Graeco-Latin square.

Notice this time how different colour-pairs are paired:

2nd row: alternate pairs
3rd row: inner and outer pairs
4th row: adjacent pairs

But experiment. What happens, for example, if the yellow-yellow, blue-blue, red-red and green-green pairs do not all occur in the same row?

Lastly we'll try something we couldn't on an order 3 square: we'll add spoons (Polydron triangles). We now require every colour in every saucer, cup and spoon row and column, and we require no cup-saucer, cup-spoon or saucer-spoon pair to be duplicated. That's why I call this task 'triple double' Sudoku.

This time I've shown the matched pairs by arrows pointing in the same direction and I've just picked out one row, the 2nd. This is how the pairing works out:

Spoon-cup:	adjacent pairs
Spoon-saucer:	inner and outer pairs
Cup-saucer:	alternate pairs

Track what happens in the other rows. Each time you'll find – you must find: otherwise there'd be a clash - all three arrangements, and in a different order each time. You might say that, in making a Graeco-Latin square, you're mixing the items up as much as possible. (It's that aspect which makes Graeco-Latin squares useful in the design of experiments which, for example, compare the effect of applying different fertilisers to a crop growing in different conditions.)

I've made all this look easy by giving you tidy solutions but I can assure you that, starting 'cold', most people spend half an hour or more solving the order 4 cup-&-saucer puzzle.

I've told you a very small part of a very long story. If you want to learn more, begin with the Wikipedia entry 'Graeco-Latin square'.

Paul Stephenson, 12.2.17