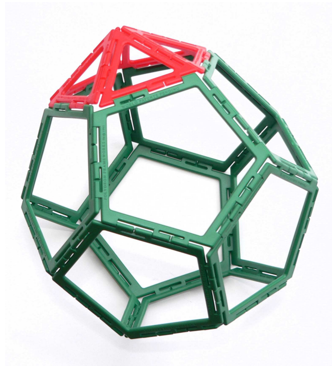
Advanced Topics - Augmenting Solids

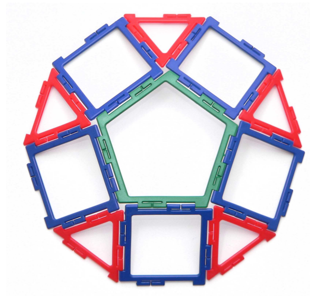
You need: 12 pentagons and 5 triangles for augmented dodecahedron;
12 decagons, 35 triangles, 3 pentagons and 12 squares for the augmented truncated dodecahedron.

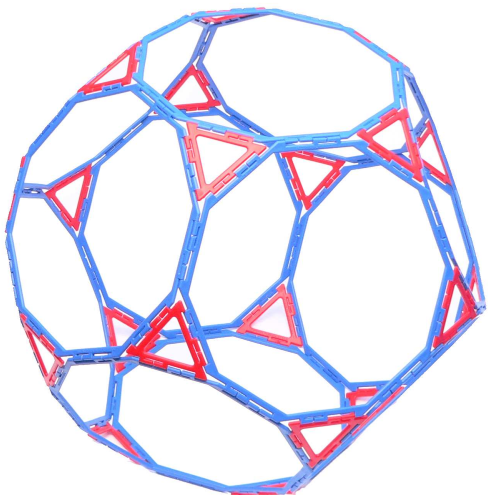
Vertex: There are several different vertices in these solids.



- ❑ To augment a solid we add sections to it. These sections are usually in the form of a cap, which replaces a single polygon with a group of polygons. Augmented solids must be convex. The solid on the right is an augmented dodecahedron.

- ❑ Try to add more caps to augment this solid. Remember that the result must be convex.

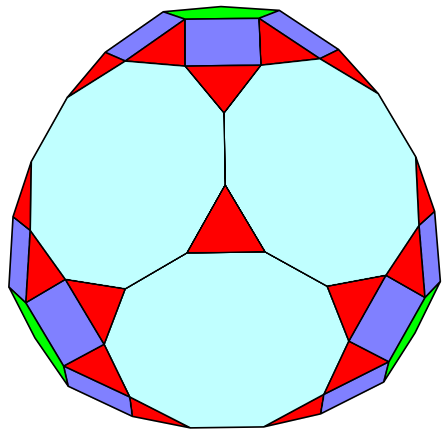




- ❑ The solid on the left is a truncated dodecahedron. Augment it by replacing a decagon with the cap or cupola shown on the right.

- ❑ Find two different ways to further augment this solid with a second identical cap of polygons.

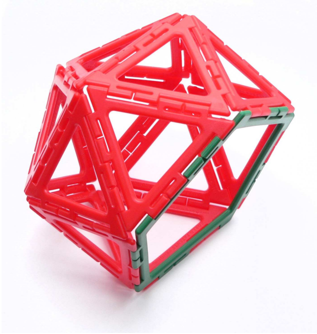
- ❑ Altogether it is possible to remove three decagons and replace each of them with caps. This is shown in the diagram on the right.



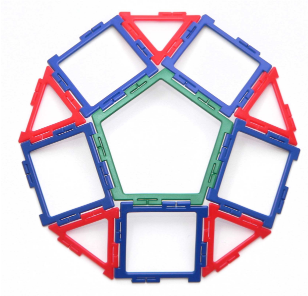
- ❑ Find which of the remaining Archimedean Solids can be augmented by removing single polygons and replacing them with caps or groups of polygons. The augmented solid must be convex.

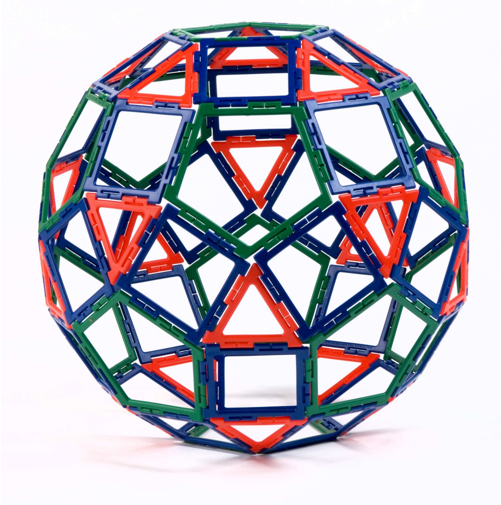
Advanced Topics - Diminishing Solids

You need: 20 triangles, 30 squares, 12 pentagons and 3 decagons .
Vertex: There are several different vertices in these solids.

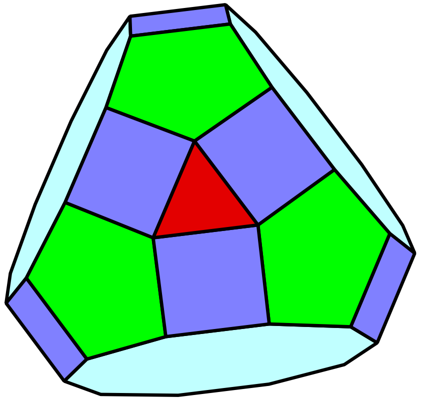


- ❑ To diminish a solid we remove a cap or group of polygons and replace those removed with a single polygon.
- ❑ In the picture on the right five triangles of an icosahedron have been replaced with a pentagon. Find several different ways to remove more triangles and replace them with pentagons.





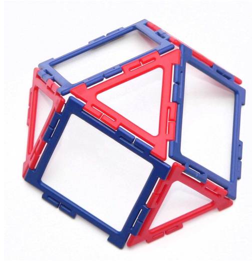
- ❑ Make the rhombicosidodecahedron (3.4.5.4) shown on the left and notice the caps like that shown on the right. Some of these caps can be removed to give different solids.
- ❑ Remove a cap and replace it with a decagon. This solid is called a diminished rhombicosidodecahedron. It could be used as a model for a geodesic dome, with the decagon as the base.
- ❑ Find two different ways to replace a second cap of this solid with a decagon.

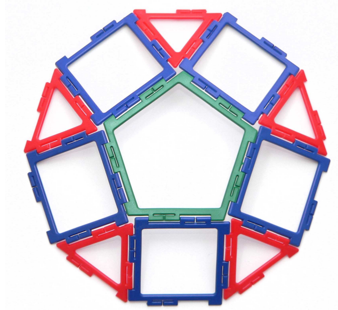


- ❑ You can remove three caps and replace each of them with decagons. This is shown in the diagram on the right.
- ❑ Find which of the remaining Archimedean Solids can be diminished by removing caps or groups of polygons, and replacing them with a single polygon.

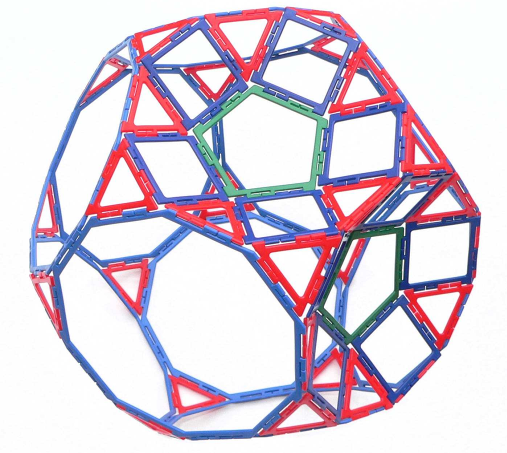
Advanced Topics – Dips and Bumps

You need: A variety of solids to experiment with

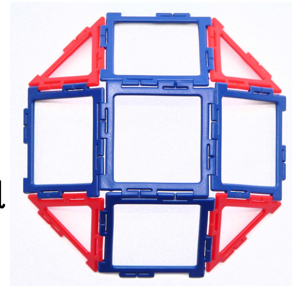




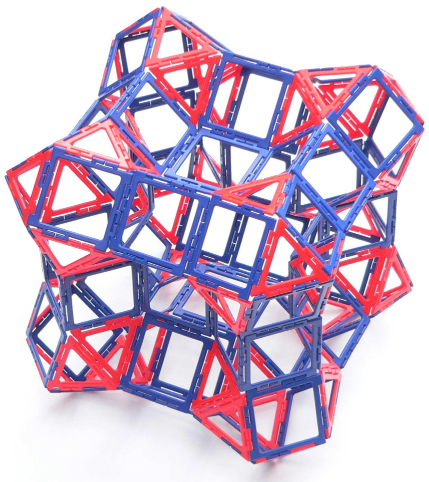
- ❑ Two of the advanced topics activities have dealt with diminishing and augmenting solids. In each case the result is a convex solid. In this activity we allow non-convex solids to be formed but we shall try to keep structure and order.



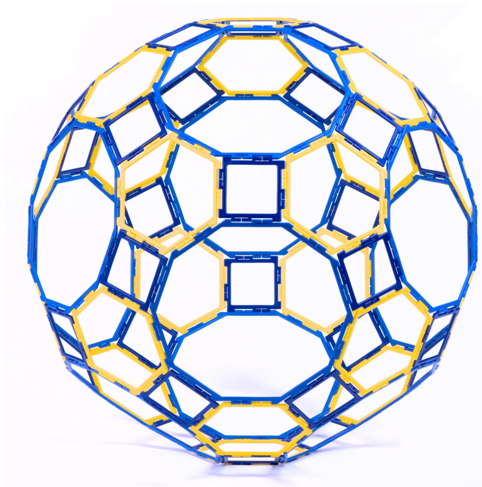
- ❑ Three 'caps' are shown on the right. These can be used to replace a hexagon, an octagon or a decagon.



- ❑ On the left two of the decagons of a truncated dodecahedron have been replaced with indented caps or 'dips'. Make this solid and replace as many of the decagons as you can with 'dips'.
- ❑ Make a truncated octahedron, with vertex notation 4.6.8. Replace the octagons with 'dips' and replace the hexagons with 'bumps'. This creates the amazing solid shown on the left.

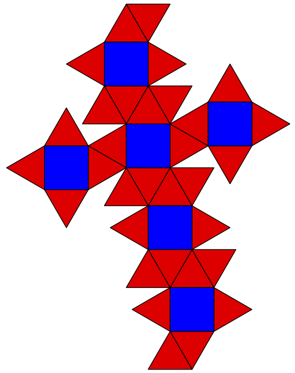


- ❑ Notice that this entire solid becomes composed of just triangles and squares.
- ❑ Build the great rhombicosidodecahedron (4.6.10) shown on the right, and replace the hexagons and decagons with 'dips' and 'bumps'.
- ❑ The caps can be placed as either 'dips' or 'bumps', but it generally looks best if one sort of cap becomes a 'dip' and the other a 'bump.



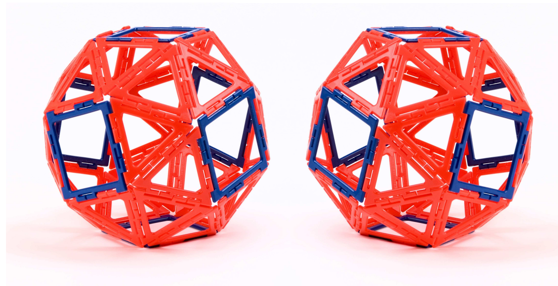
Advanced Topics - Left-handed and Right-handed Solids

You need: 48 pentagons, 12 squares and lots of triangles



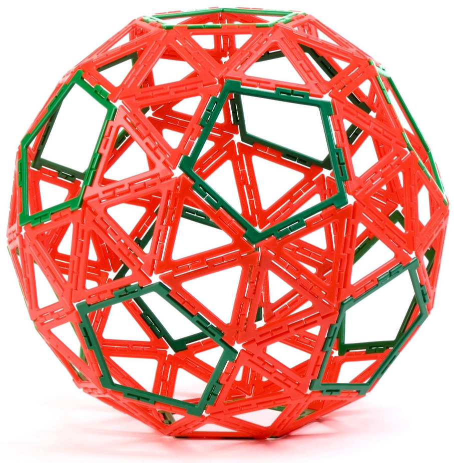
- ❑ Symmetry is an important property which mathematicians use to describe an object, and to learn more about it. Of the thirteen Archimedean Solids, eleven have planes of symmetry. The two which do not are the snub cube and the snub dodecahedron.

- ❑ Each of the two solids without planes of symmetry exist in two forms, one is left-handed and the other right-handed. Solids with this property are called chiral.

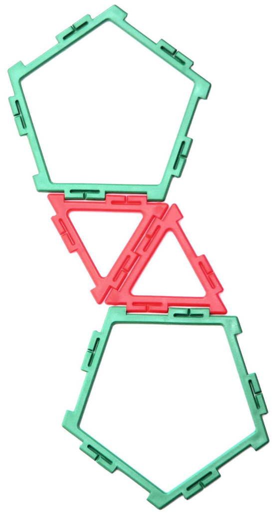


- ❑ Make two copies of the net for a snub cube, shown above. Now fold the edges of one net upwards towards you, to make the left-hand solid of the pair shown. Fold the edges of the other net down and away from you, to make the other solid.

- ❑ Convice yourself that no matter how you rotate and move one solid you cannot make it look identical to the other one. Arrange the two solids so that you can see that one is a reflection of the other.

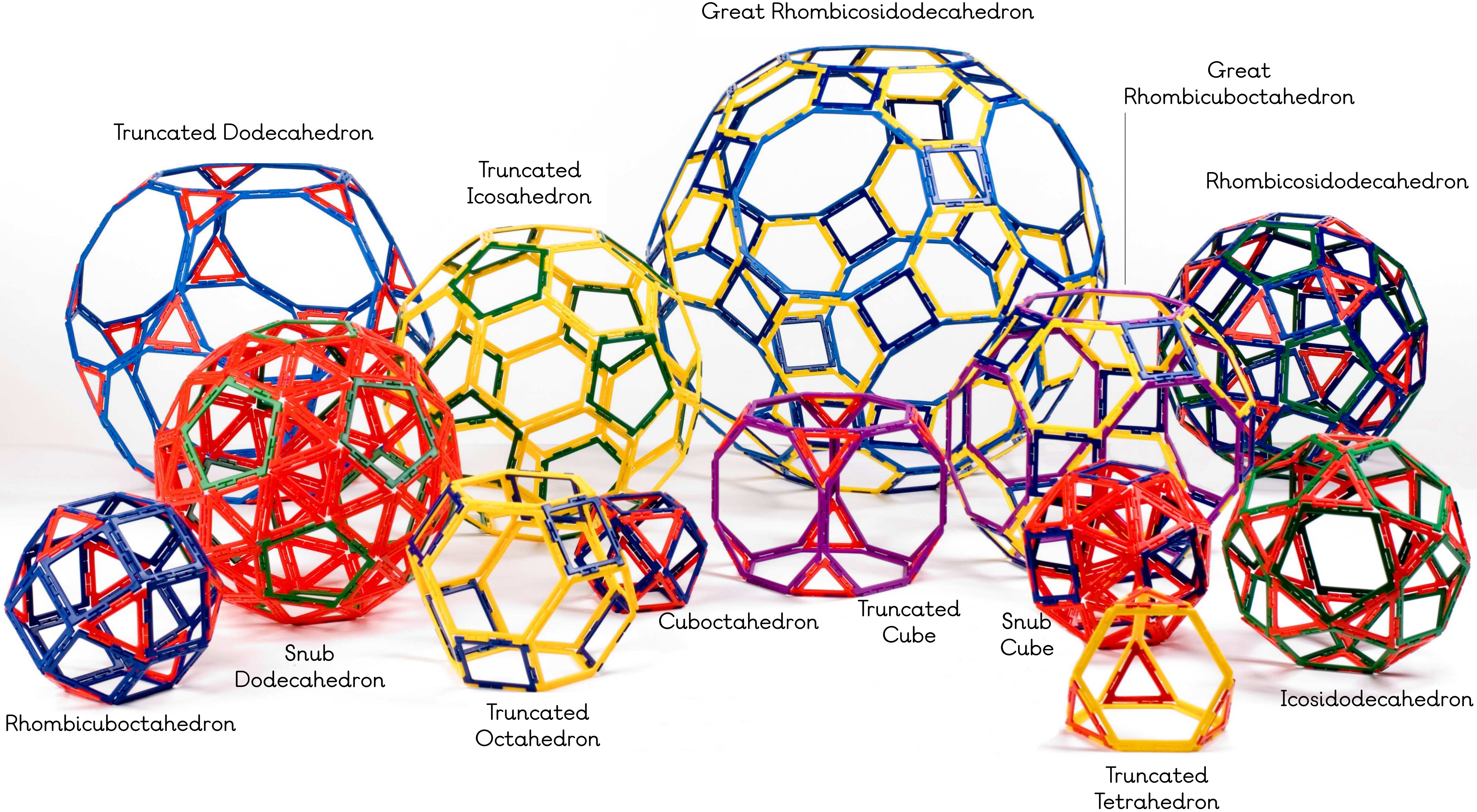


- ❑ The solid on the left is the snub dodecahedron. To make it begin with the partial net of two pentagons and two triangles, shown on the right. Add a ring of triangles around each pentagon. Fold this partial net down and away from you, and then complete the solid.



- ❑ Repeat this process with the other partial net, but this time fold it up and towards you. Arrange the two solids so that you can see that one is a reflection of the other.

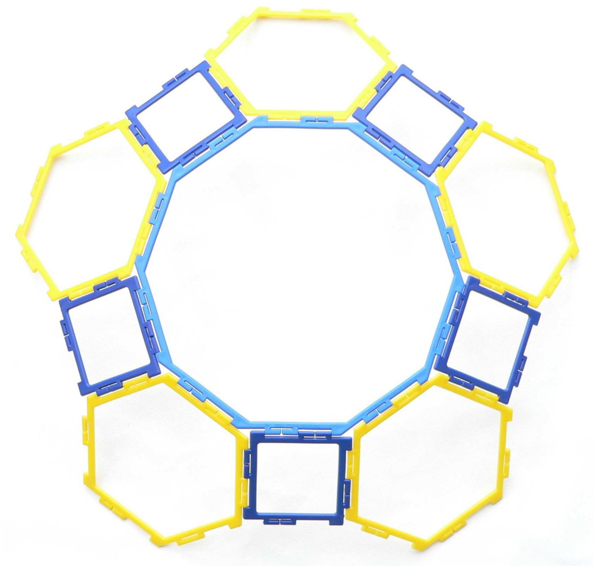
The Family of Archimedean Solids

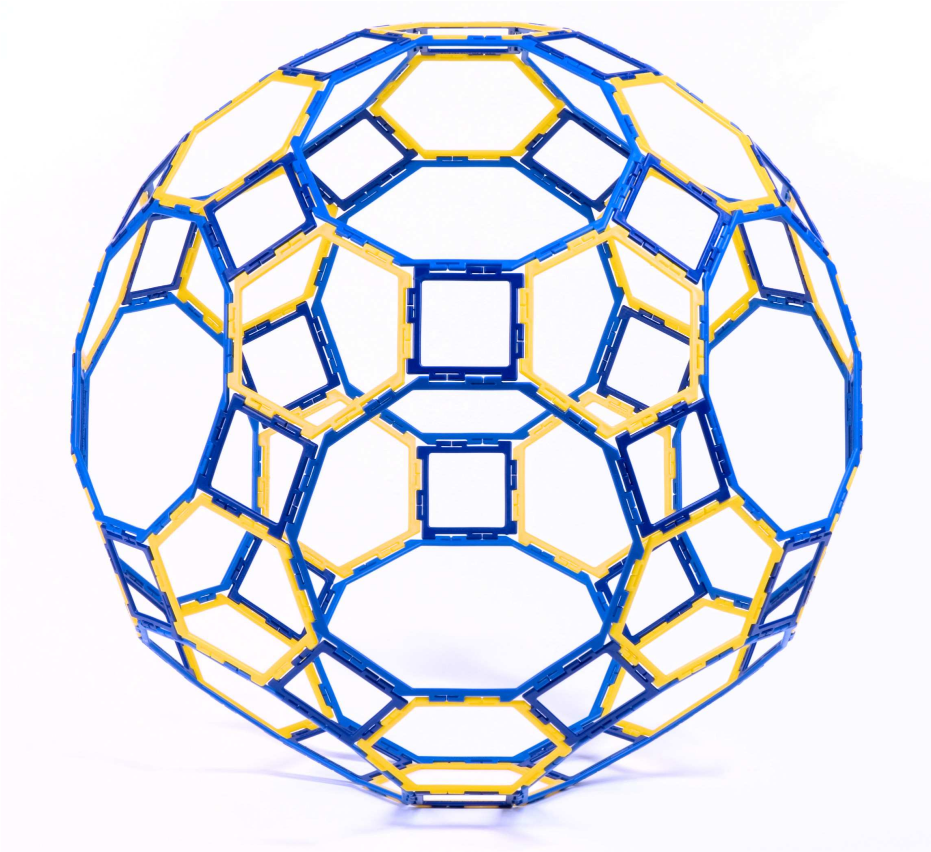


Great Rhombicosidodecahedron

You need: 12 decagons, 20 hexagons and 30 squares
Vertex: 4.6.10

- ❑ This is a wonderful solid, and is the largest of the Archimedean Solids.
- ❑ To make it, first begin with the bowl shown on the right. Notice that every vertex is 4.6.10.

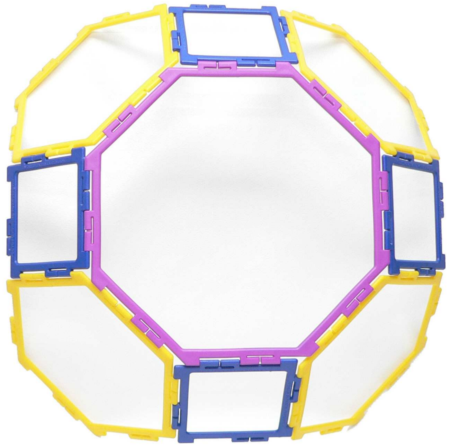




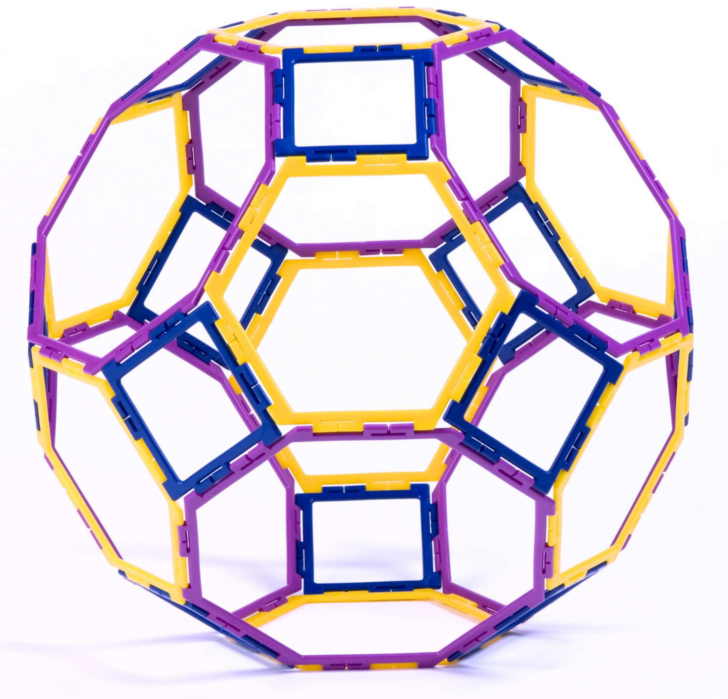
- ❑ Continue to build the solid, making sure that every vertex is 4.6.10.
- ❑ A good idea with this solid is to focus on the decagons, and make sure they are always surrounded with alternate squares and hexagons.
- ❑ Can you find the number of vertices this solid has, without counting them all?

Great Rhombicuboctahedron

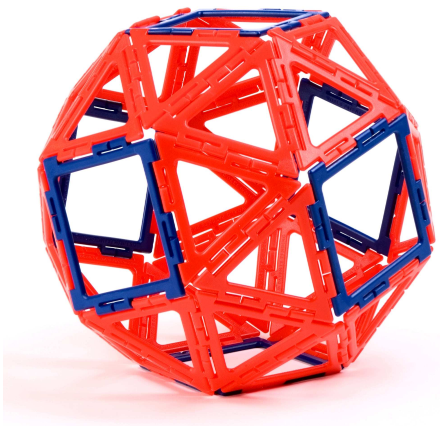
You need: 6 octagons, 8 hexagons and 12 squares.
Vertex: 4.6.8



- ❑ This solid straightforward and very attractive.
- ❑ Start with the cap shown on the right. Notice that this is an octagon with a ring of alternating squares and hexagons.
- ❑ Continue to construct this solid, making sure that each vertex has the form 4.6.8. This means that one square, one hexagon and one octagon meet at each vertex.

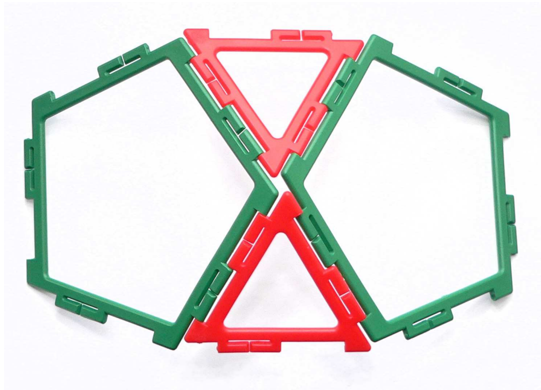


- ❑ This solid has lots of symmetry. Can you work out how many planes of symmetry the solid has? A plane of symmetry cuts the solid into two so that each piece is the mirror image of the other.
- ❑ To tackle this problem, it might be a good idea to place it on a surface and to use very long rubber bands or string, to mark each plane.
- ❑ In contrast, the solid shown on the right has no planes of symmetry. Can you find another Archimedean Solid with no planes of symmetry?

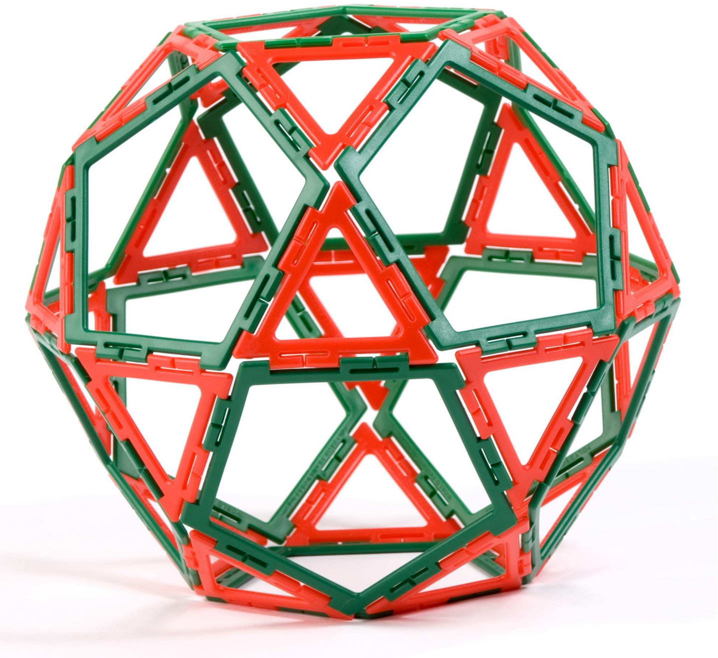


Icosidodecahedron

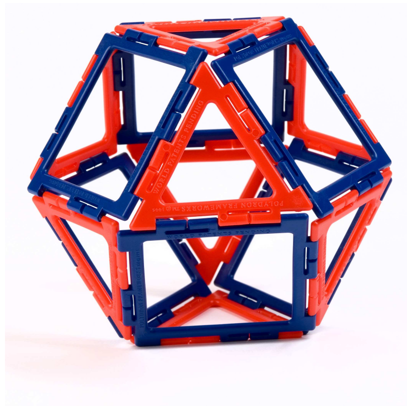
You need: 12 pentagons and 28 triangles and 6 squares
Vertices: 3.5.3.5 for icosidodecahedron and 3.4.3.4 for cuboctahedron



- ❑ This is a straightforward solid to make if you observe a simple rule. Every pentagon is surrounded by triangles and every triangle is surrounded by pentagons.
- ❑ Start with the cap shown on the right and then follow the rule given above.
- ❑ An unusual feature of this solid can be seen in the picture on the left below. It can be separated into two identical halves along an 'equator'.

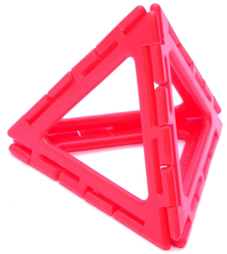


- ❑ Break the solid into two halves and re-connect it so that the pentagons connect to pentagons and triangles connect to triangles. Can you explain why this is no longer an Archimedean Solid?
- ❑ Look at the cuboctahedron, shown on the right. Describe the similarities between this and the icosidodecahedron.
- ❑ What happens if you replace the pentagons in the icosidodecahedron with hexagons?
- ❑ Count the pieces in each of these solids and use this information to explain their names.

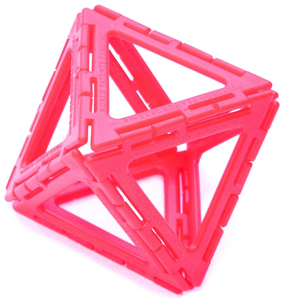


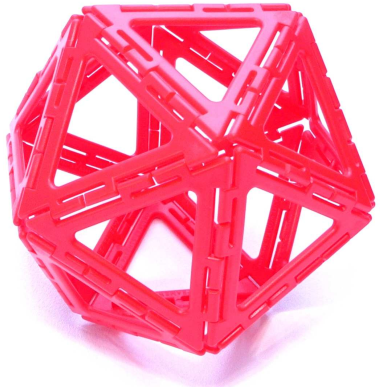
Platonic Solids

You need: 32 triangles 6 squares and 12 decagons.

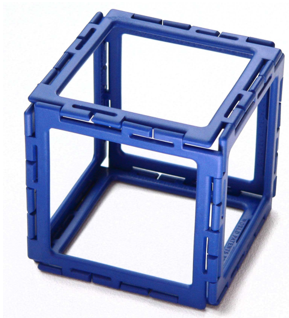


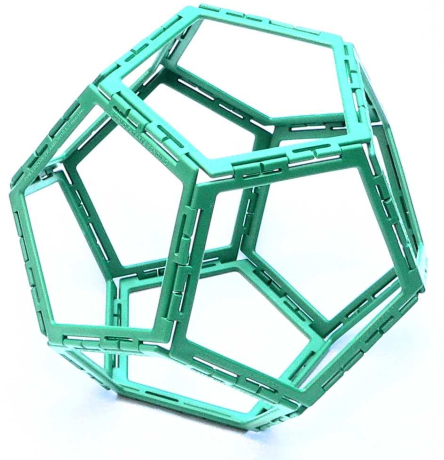
- ❑ To understand the Archimedean Solids we first need to understand the Platonic Solids. These five solids have the property of vertex regularity, but each solid is made from only one sort of regular polygon.
- ❑ The simplest of the solids is the tetrahedron, made from four triangles. In the picture above you can see that it has three triangles meeting at each vertex.





- ❑ Make a a solid with four triangles meeting at each vertex. You will have an octahedron, as shown on the left.
- ❑ If we allow five triangles to meet at each vertex, we make an icosahedron, as shown on the right.
- ❑ What happens if we allow six triangles to meet at each vertex?

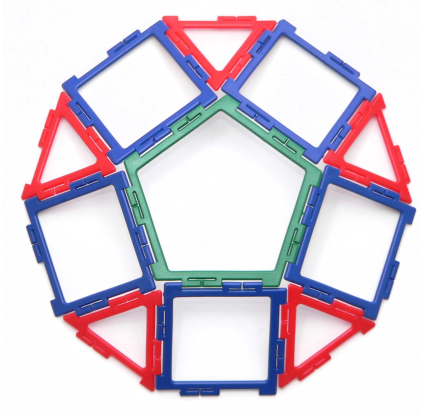




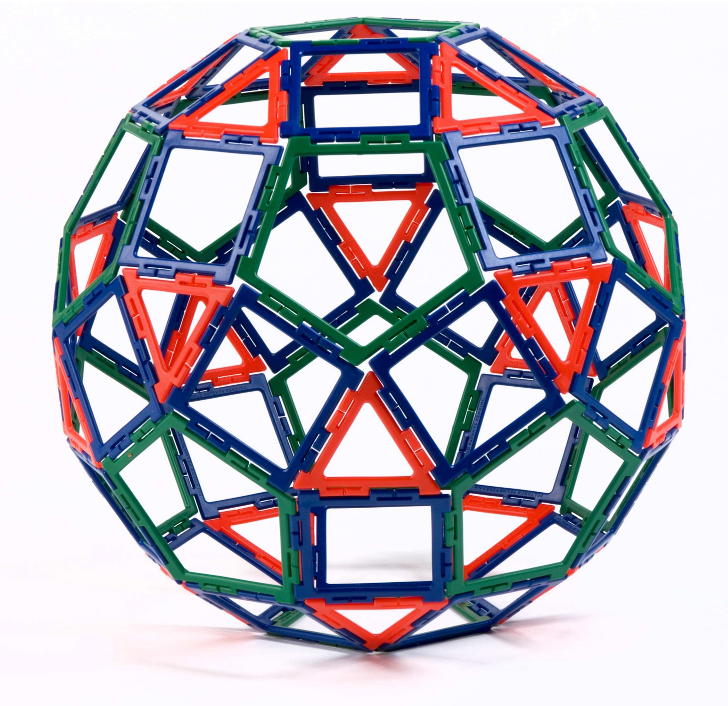
- ❑ If we use only squares, three of them meet at each vertex, and we have a cube. What happens if we put four squares at each vertex?
- ❑ Take twelve pentagons and make the dodecahedron, with three pentagons at each vertex.
- ❑ Explain why no other solids can be made with only one sort of regular polygon, where every vertex is the same.

Rhombicosidodecahedron

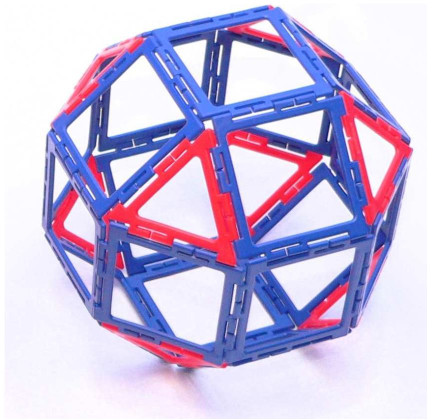
You need: 20 triangles, 30 squares and 12 pentagons.
Vertex: 3.4.5.4



- ❑ This is possibly the most complex of the Archimedean Solids. At each vertex four polygons meet; two squares, a triangle and a pentagon.
- ❑ Start with the cap shown on the right. Notice that the pentagon is surrounded by a ring of alternating squares and triangles, with squares connected to the pentagon.
- ❑ Make sure that you arrange the shapes in the correct order. The squares do not touch each other, and every vertex must have the arrangement 3,4,5,4.

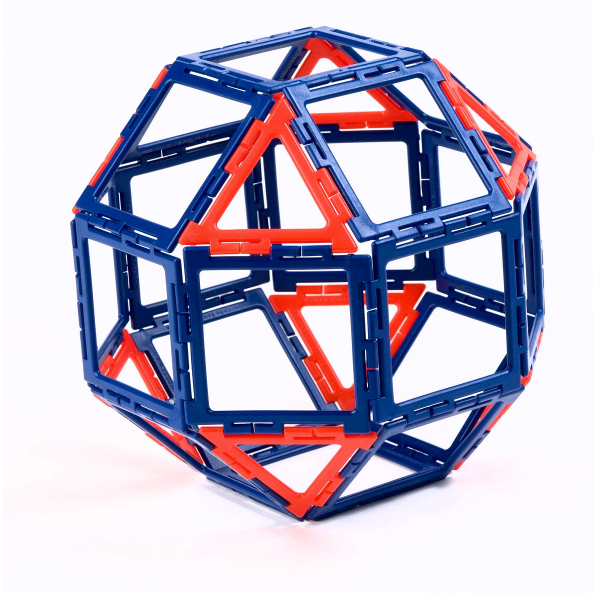


- ❑ To explore the symmetry of this solid, place it on the floor with a square at the top and the bottom. Look directly down from above and describe the symmetry you see.
- ❑ Repeat this activity but with a triangle or a pentagon on the top and the bottom.
- ❑ Imagine replacing each pentagon with a square. You will have the smaller solid shown on the right.
- ❑ Replace each pentagon with a hexagon and make the semi-regular tessellation with vertex arrangement 3.4.6.4.

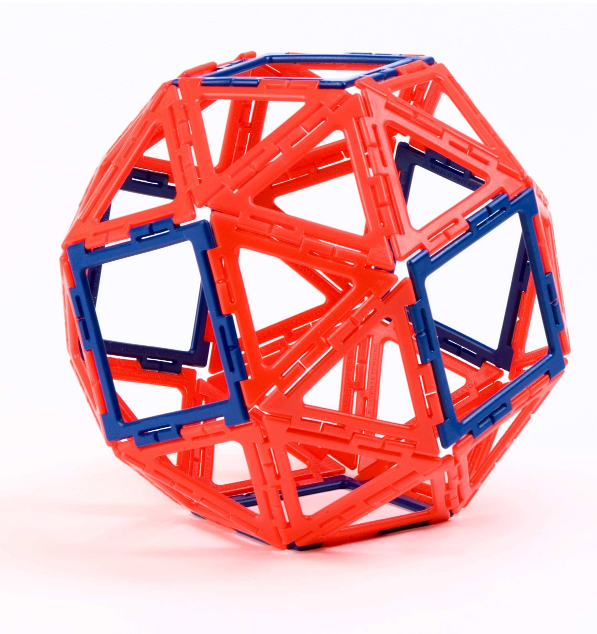


Similarities and Differences

You need: Lots of squares and triangles
Vertices: 3.3.3.3.4 and 3.4.4.4



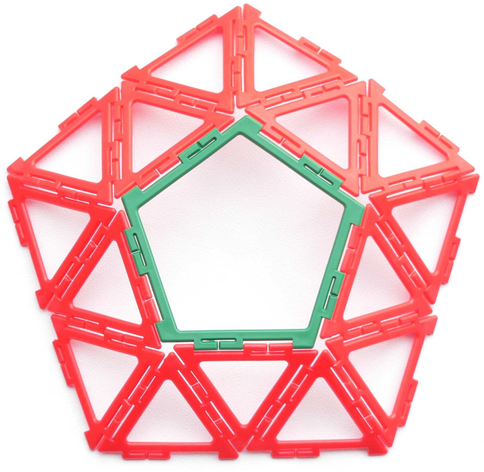
- ❑ The two solids shown on this page have some similarities. In the solid on the right, every triangle is surrounded by squares, and in the one below, every square is surrounded by triangles.
- ❑ The solid on the right is the rhombicuboctahedron. Every vertex is 3.4.4.4.
- ❑ There is more than one way to make this solid. In the picture notice that there are two belts of squares which pass all the way around the solid.



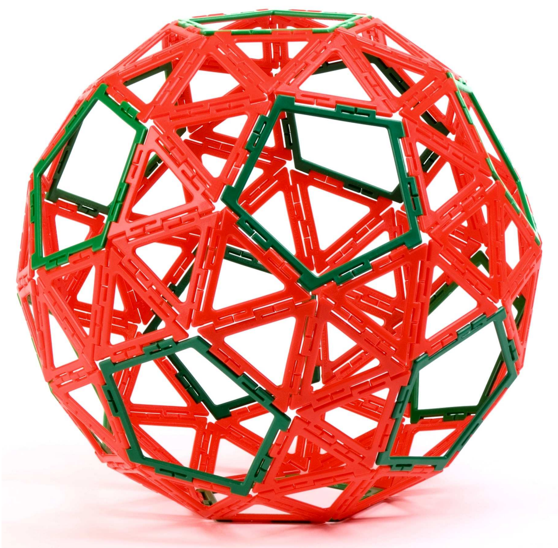
- ❑ The solid on the left is the snub cube. It has vertex notation 3.3.3.3.4. There are two ways to make this solid but it is very hard to tell them apart.
- ❑ Find a vertex on each solid and work out the sum of the angles which meet there. What do you notice?
- ❑ Find out how many planes of symmetry each of the solids has. The result may surprise you, and illustrates one way in which the solids are very different. One way to show a plane of symmetry is to use a large rubber band, wrapped around the solid.
- ❑ Make another Archimedean Solid using only triangles and squares. Can you explain why there are no more than these three which have only squares and triangles?

Snub Dodecahedron

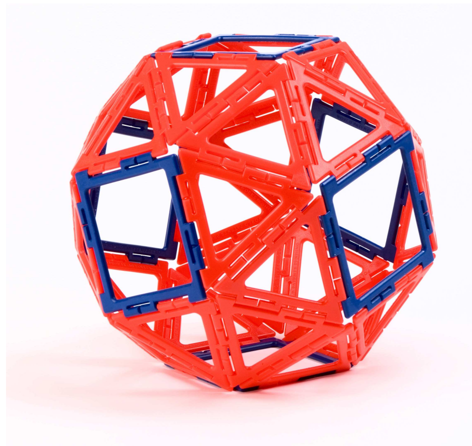
You need: 12 pentagons and 80 triangles
Vertex: 3.3.3.3.5



- ❑ This solid looks straightforward but is quite tricky to make.
- ❑ Start with the cap shown on the right. Notice that this is a pentagon completely surrounded with triangles.
- ❑ As you continue to make this solid, make sure that you always have a ring of triangles around every pentagon.
- ❑ It may seem a good idea to make lots of caps like the one above and then connect them together. Unfortunately, this idea will lead to problems with triangles overlapping.

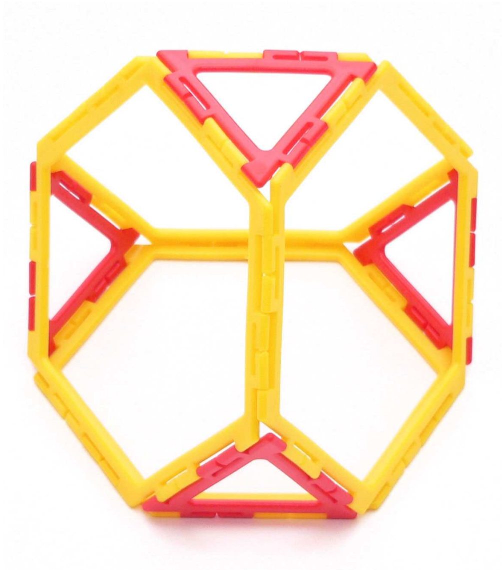


- ❑ It is interesting to compare this solid with the snub cube (3.3.3.4), shown below. In one there are rings of triangles around a pentagon, in the other there are rings of triangles around a square.
- ❑ Describe what happens when you try to make a 'new' Archimedean Solid, with rings of triangles around a triangle or rings of triangles around a hexagon.
- ❑ Try to describe any symmetry in the snub dodecahedron.

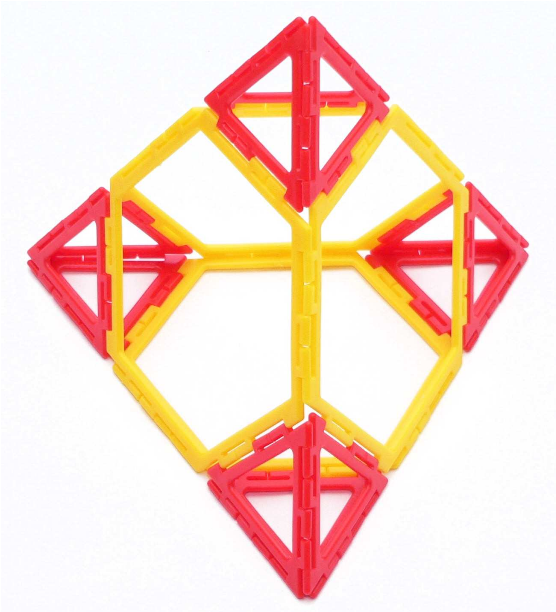


Truncating Solids 1

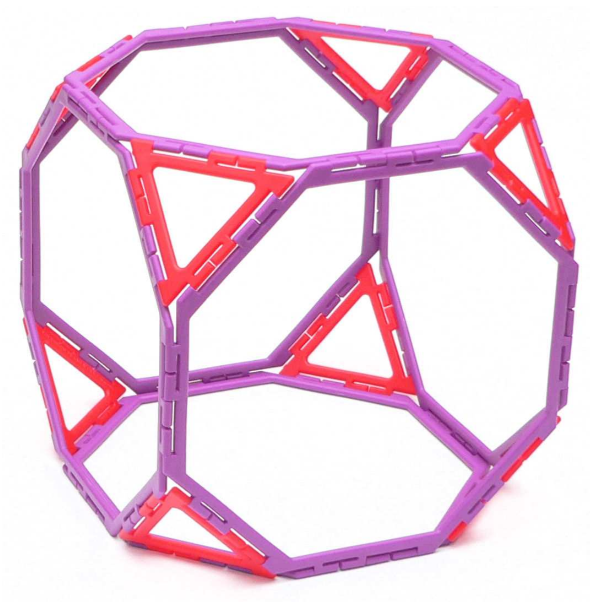
You need: 24 triangles, 8 hexagons and 6 octagons.
Vertices: 3.6.6 and 3.8.8



- ❑ To truncate a solid you need to slice through or cut off each vertex of the solid, to make a new one. The slicing must be done in a special way.
- ❑ Make the truncated tetrahedron shown on the right. Each vertex is 3.6.6.
- ❑ This solid is called a truncated tetrahedron because you can start with a large tetrahedron, divide each edge into three, and cut off each vertex, one third of the way along an edge.
- ❑ Make the the tetrahedron shown below. Truncate it by removing each of the vertices.

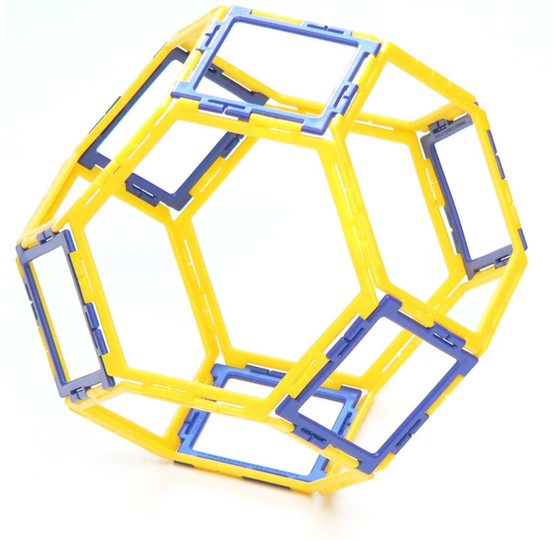


- ❑ Make a large tetrahedron, like the one on the left, but made entirely from small triangles. Each triangular face is made from nine small triangles.
- ❑ Truncate your large tetrahedron.
- ❑ On the right is a truncated cube.
- ❑ Build this solid and explain how a cube has been truncated to make it.

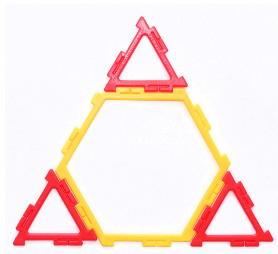


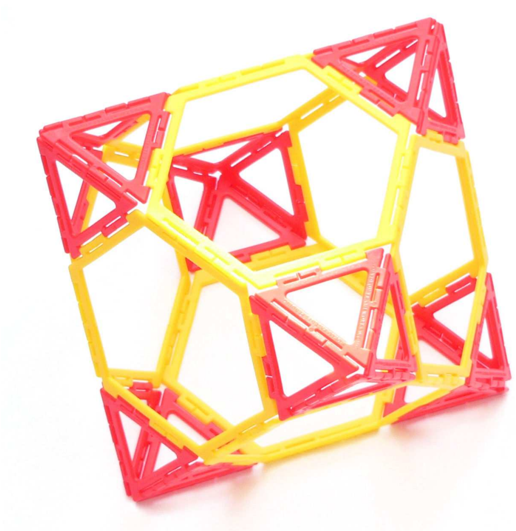
Truncating Solids 2

You need: 32 triangles, 8 hexagons and 6 squares.
Vertex: 4.6.6

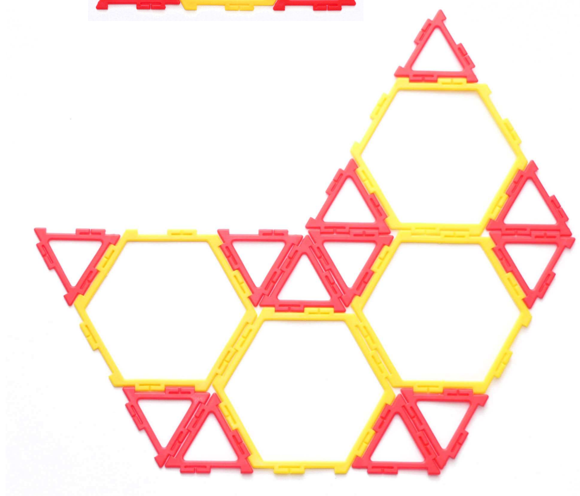


- ❑ To truncate a solid you need to slice through or cut off each vertex of the solid to make a new one. The slicing must be done in a special way.
- ❑ Make the truncated octahedron shown on the right. Each vertex is 4.6.6.
- ❑ This solid is called a truncated octahedron because you can start with a large octahedron, divide each edge into three, and cut off each vertex one third of the way along an edge.
- ❑ Make the large octahedron shown below from large triangles, shown on the right. One way to do this is to make two copies of the net shown on the right.



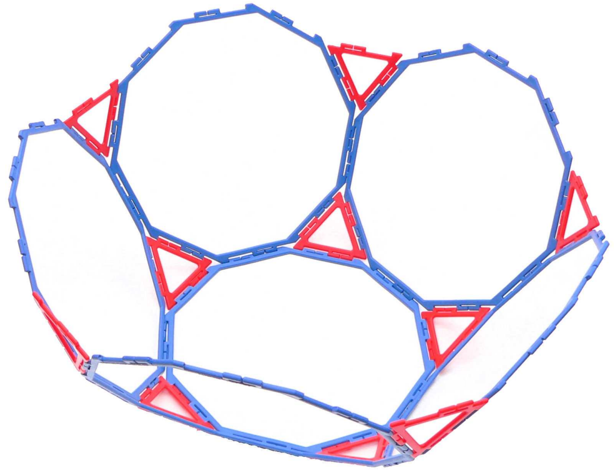


- ❑ Truncate your octahedron by removing a pyramid from each of the vertices.
- ❑ Notice that as each vertex of the octahedron is cut, it becomes a square, and each triangle doubles the number of its edges, to become a hexagon.
- ❑ Try to make a truncated octahedron by using only small triangles and squares.

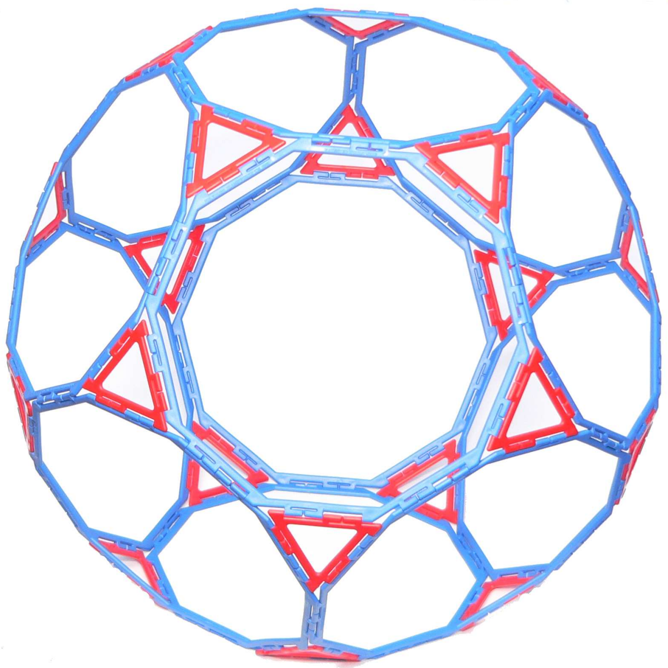


Truncating Solids 3

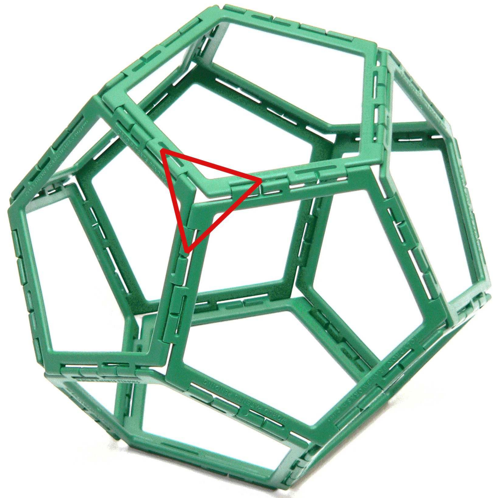
You need: 20 triangles, 12 decagons and 12 pentagons.
Vertex: 3.10.10



- ❑ To make a truncated dodecahedron, first make the the bowl shown on the right. This needs half of the pieces.
- ❑ You can complete the solid by making an identical bowl and the joining them together. To ensure that each bowl will fit together, make sure that the embossed writing on each piece is on the inside of the solid.



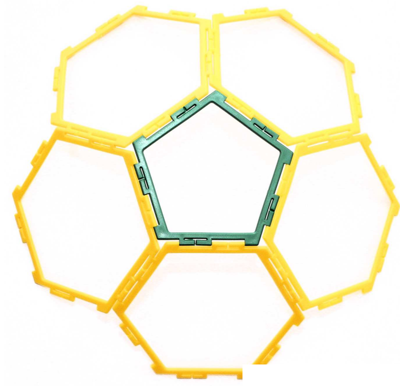
- ❑ The completed solid has a lot of rotational symmetry. This can be seen in the picture on the left.
- ❑ To understand how the solid gets its name, we can think about truncating the dodecahedron below. The red triangle shows how we cut off a vertex to leave a triangle.



- ❑ Each vertex of the dodecahedron is truncated to make a triangle, and each pentagon becomes a decagon, by doubling the number of edges.
- ❑ Use the information above to calculate how many triangles and decagons there are in this solid.

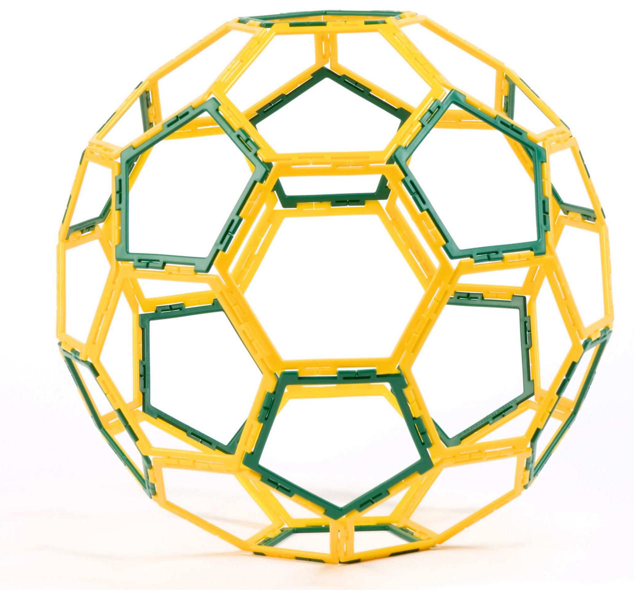
Truncating Solids 4

You need: 20 hexagons, 12 pentagons and 20 triangles.
Vertex: 5.6.6



- ❑ The solid shown below is called a truncated icosahedron. To make it, first construct the the bowl shown on the right. This has one pentagon surrounded by five hexagons.

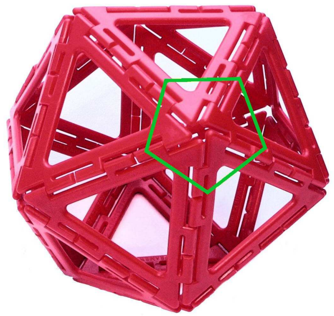
- ❑ There are two simple rules or tips to follow, if you have difficulty with this solid. First make sure that pentagons never touch each other. Second, never allow three hexagons to meet at a vertex.



- ❑ The completed solid may remind you of a football. Some footballs use a patchwork of hexagons and pentagons identical to this solid.

- ❑ To understand how the solid gets its name, we can think about truncating the icosahedron below, by cutting off each of its twenty vertices. The green pentagon shows you how this might look.

- ❑ Once each vertex of the icosahedron has been truncated to make a pentagon, each existing triangle will have double the number of edges, and become a hexagon.



- ❑ Use the information above to calculate how many hexagons and pentagons there are in a truncated icosahedron.